260. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

The necessary and sufficient condition that a binary form be a perfect nth power is that its Hessian vanish.

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting $\frac{\partial u}{\partial x}$ by p, $\frac{\partial u}{\partial y}$ by q, the vanishing of the Hessian shows that $p=f(q)$, *i. e.*, $q=mp$, since both p and q are homogeneous and of the same degree. By Lagrange's method of solving partial differential equations, we have

$$\frac{dx}{m}=\frac{dy}{-1}=\frac{du}{0}.$$

Hence, u=constant, $x+my$=constant, and a general solution is given by

$$u=f(x+my)=(x+my)^n,$$

since u is homogeneous in x, y. It is easily verified that when $u=(x+my)^n$ the Hessian vanishes. Hence this condition is both necessary and sufficient.

II. Solution by the PROPOSER.

A slightly different point of view from the above is afforded by the following method:

The Hessian is the Jacobian of the first derivatives p and q. Hence $p-mq=0$. Also $xp+yq=nu$, n being the order of u. Solving for p and q,

$$p=\frac{nmu}{y+mx},\quad q=\frac{nu}{y+mx}.$$

Also, $du=pdx+qdy=nu\frac{dy+mdx}{y+mx}$, or $\frac{du}{u}=n\frac{d(y+mx)}{y+mx}$.

Henge, $\log u=n \log k(y+mx)$, $u=(a_1x+a_2y)^n$.

261. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum to infinity the series, $\frac{1}{n^p}+\frac{3}{n^{2p}}+\frac{5}{n^{3p}}+\frac{7}{n^{4p}}+\frac{9}{n^{5p}}+\dots\dots\dots$

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting n^{-p} by x, we have

$$\sum_{i=1}^{\infty}\frac{(2i-1)}{n^{ip}}=x[1+3x+5x^2+7x^3+\dots\dots\dots]$$
$$=x\sum(2r+1)x^r=2x\sum rx^r+x\sum x^r$$